Robust Secret Sharing Schemes Against Local Adversaries

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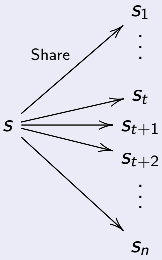
March 9, 2016

Threshold Secret Sharing

(Share, Rec) pair of algorithms:



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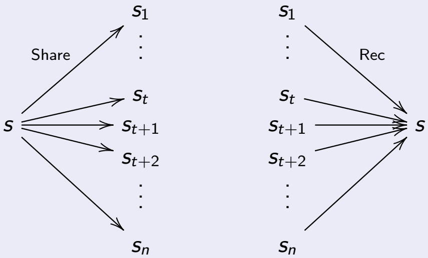

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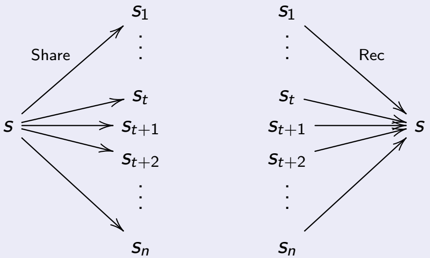

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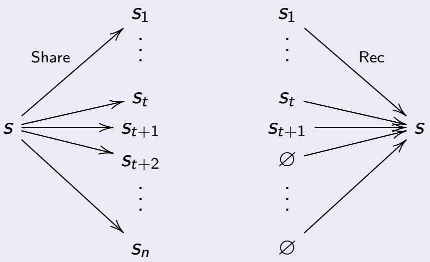

***t*-privacy:** $s_1, \ldots, s_t \Rightarrow$ no info on s

$(t+1)$-reconstructability: $s_1, \ldots, s_{t+1} \Rightarrow$ s uniquely determined

Example: Shamir Secret Sharing [Sha79]

Shamir.Share$_t(s)$:

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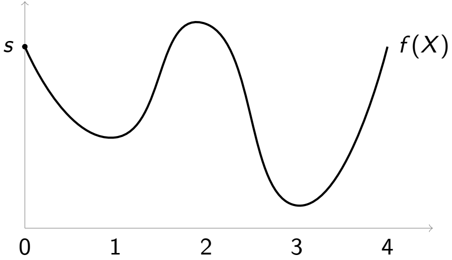
1. sample uniform polynomial $f(X)$ with
   - degree t
   - $f(0) = s$



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Shamir.Share$_t(s)$:

1. sample uniform polynomial $f(X)$ with
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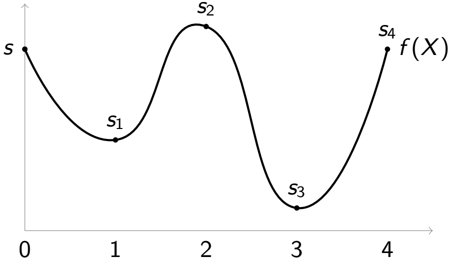
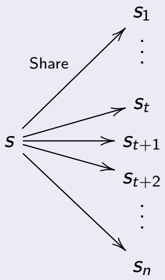
2. compute $s_i \leftarrow f(i)$
3. output $(s_1, \ldots, s_n)$



Robust Secret Sharing – Standard Model

(Share, Rec) Secret Sharing, (t, δ)-**robust**: for any Adv,



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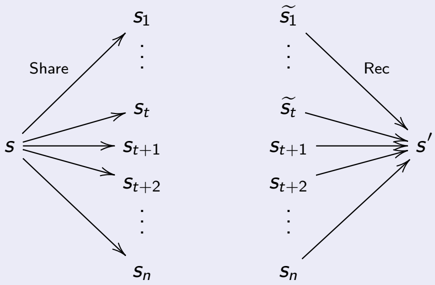
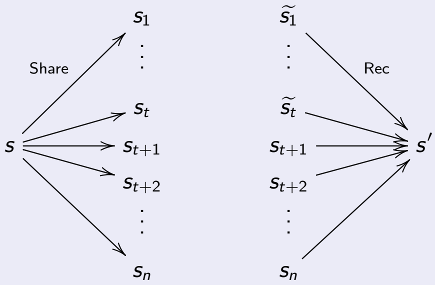


$$\Pr[s' \neq s] \leq \delta \qquad \text{where} \qquad (\widetilde{s}_1, \ldots, \widetilde{s}_t) = \mathsf{Adv}(s_1, \ldots, s_t)$$

Robust Secret Sharing – with Local Adversaries

(Share, Rec) Secret Sharing, (t, δ)-**robust**: for any $\mathsf{Adv}_1, \ldots, \mathsf{Adv}_t$,



$$\Pr[s' \neq s] \leq \delta \qquad \text{where} \qquad \widetilde{s}_i = \mathsf{Adv}_i(s_i)$$

Why Locality? – Possible Scenarios

- Corrupt parties unwilling to coordinate (e.g. different goals)
- Corrupt parties oblivious about existence of each other
- Network with (independently) faulty channels
- Data is required to travel fast, coordination impossible
- ...

Locality – Related Work

Interactive Proofs:

- Multi-prover interactive proofs:
  MIP=NEXP, [BFL91] (IP=PSPACE, [Sha92])

Multi-party Computation:

- Collusion-free protocols [LMs05, AKL$^+$09, AKMZ12]
- Local UC [CV12]

Leakage-resilient crypto:

- Split secret state and independent leakage [DP08]

Facts about Robust Secret Sharing

$t < n/3$: perfect robustness ($\delta = 0$)

no share size overhead ($|s_i| = |s| =: m$)

e.g. Shamir share + Reed-Solomon decoding

RS decodes up to $(n - t)/2 > (3 \cdot t - t)/2 = t$ errors

$n/3 \leq t < n/2$: tricky!

no perfect robustness ($\delta = 2^{-k}$) [Cev11]

shares larger than secret ($|s_i| > m$) [Cev11]

All of the above: independent of standard/local adv. model

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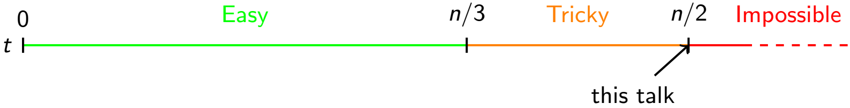

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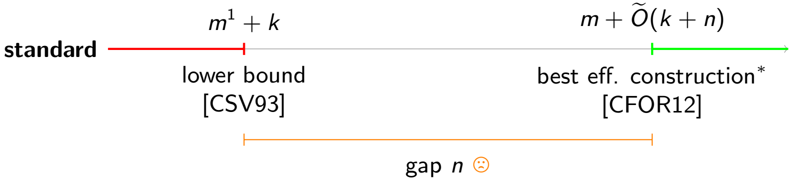
The Trickiest Case: $n = 2 \cdot t + 1$

Analysis of $|s_i|$:



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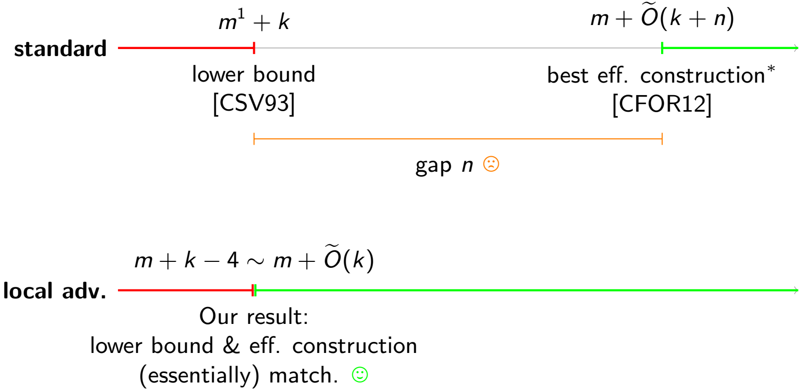

[1] m = message length

Our Construction

Previous Constructions

Privacy: Shamir secret sharing, degree=t

Robustness: one-time MAC, $O(n)$ keys per player.

$\Rightarrow |s_i|$ inherent depends (at least) linearly on n

Our Construction

Privacy: Shamir secret sharing, degree=t

Robustness: one-time MAC, one key only.

In Detail

Share(s):

1. sample MAC key $z \in X$
2. $(s_1, \ldots, s_n) \leftarrow \text{Shamir.Share}_t(s)$
3. $(z_1, \ldots, z_n) \leftarrow \text{Shamir.Share}_1(z)$
4. $t_i \leftarrow \text{MAC}_z(s_i)$
5. output $S_i = (s_i, z_i, t_i)$ to P_i

Rec($S_1, \ldots, S_n$):

1. $z \leftarrow \text{Shamir.Rec}_1(z_1, \ldots, z_n)$
2. set $i \in G$ if $t_i = \text{MAC}_z(s_i)$
3. $s \leftarrow \text{Shamir.Rec}_t(s_G)$

Privacy – Proof Intuition

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***t*-privacy:** z uniform, independent of $s, s_1, \dots, s_n$
$s_1, \dots, s_t$ give no info on s, (privacy of Shamir.Share_t)
$t_1, \dots, t_t$ functions only of $z, s_1, \dots, s_t$
$\Rightarrow S_1, \dots, S_t$ give no info on s

Robustness – Proof Intuition

$\mathrm{Rec}(S_1, \ldots, S_n)$:

1. $z \leftarrow \mathrm{RS.Rec}_1(z_1, \ldots, z_n)$
2. set $i \in G$ if $t_i = \mathrm{MAC}_z(s_i)$
3. $s \leftarrow \mathrm{Shamir.Rec}_t(s_G)$

(t, δ)-robustness: z correct, because $\mathrm{RS.Rec}_1$ decodes up to $(n-1)/2 = (2t+1-1)/2 = t$ errors

Adv_i **sees only** s_i, z_i, t_i
$\Rightarrow$ no info on z (privacy of $\mathrm{Shamir.Share}_1$)

MAC ε-secure
$\Rightarrow \Pr[i \in G \mid \widetilde{s}_i \neq s_i] \leq \varepsilon$
$\Rightarrow \Pr[G \subseteq H \cup P] \geq 1 - t \cdot \varepsilon$
$\Rightarrow \delta \leq t \cdot \varepsilon$

Possible MAC and Overhead Analysis

Remember: $\delta \leq t \cdot \varepsilon$
Assume: $m = |s|, \qquad 2 \cdot c = |z|, \qquad c = |t_i|, \qquad m = 2 \cdot d \cdot c$

$$\begin{array}{rccccl} \text{MAC}: & (\mathbb{F}_{2^c})^2 & \times & \mathbb{F}_{2^m} & \rightarrow & \mathbb{F}_{2^c} \\ & (a, b), & & (m_1, \ldots, m_d) & \mapsto & \sum_{l=1}^{d} a^l \cdot m_l + b. \end{array}$$

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Set $c = k + \log(t \cdot m) = \widetilde{O}(k) \Rightarrow \delta \leq t \cdot m \cdot 2^{-k-\log(t \cdot m)-1} \cdot c^{-1} \leq 2^{-k}$

Overhead: $|z| + |t_i| = 2c + c = 3c = \widetilde{O}(k)$

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Want to show:

Scheme $(t, 2^{-k})$-robust against local advs $\Rightarrow |s_i| \geq m + k - 4$

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What we do: prove a stronger result!

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local adv: $\widetilde{s}_i = \mathrm{Adv}_i(s_i)$

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Proof structure:

1. define an oblivious attack
2. link success of attack with share size

The Attack

Let s_{t+1} be the shortest share.

Specifications:

- "decide" who to corrupt: $P_1, \ldots, P_t$ (L) or $P_{t+2}, \ldots, P_n$ (R)
- sample secret $\widetilde{s}$, randomness $\widetilde{r}$
- run $(\widetilde{s_1}, \ldots, \widetilde{s_n}) \leftarrow \text{Share}(\widetilde{s}, \widetilde{r})$
- if L, submit $\widetilde{s_1}, \ldots, \widetilde{s_t}$; if R, submit $\widetilde{s_{t+2}}, \ldots, \widetilde{s_n}$

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Intuition: hope that corrupt shares & s_{t+1} consistent with dishonest secret.

$$\text{Rec}\left(\overbrace{s_1, \ \ldots, \ s_t, \ s_{t+1}}^{\text{partial sharing of } s^L}, \ s_{t+2}, \ \ldots, \ s_n \right) = ?$$

where $s_{t+1}, s_{t+2}, \ldots, s_n$ is the partial sharing of s^R.

Who to Corrupt?

Intuitively: find out whether L is more promising than R.

- Graph: (s^L, r^L) connected to (s^R, r^R) if:
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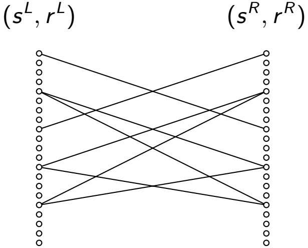

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- Label edge with L (resp. R) if:
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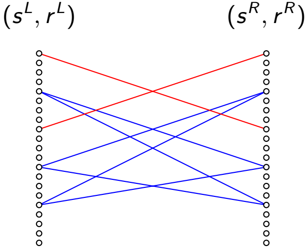

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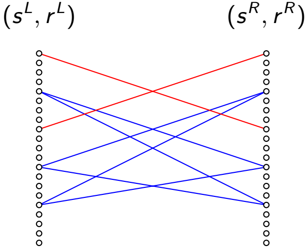
- Decide L if #L-edges $\geq$ #R-edges.



Success Evaluation (WLOG assume L)

$$\overbrace{\underbrace{s_1,\ \ldots,\ s_t}_{\widetilde{s}},\ s_{t+1}}^{s^L},\ \underbrace{\phantom{s_{t+1}}}_{}$$

$$\overbrace{\underbrace{s_1, \ldots, s_t}_{\widetilde{s}},\ s_{t+1}}^{s^{L}}, s_{t+2}, \ldots, s_n \quad \text{with } s^{R} = s_{t+1}, s_{t+2}, \ldots, s_n$$

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$$\text{Rec}\left(\overbrace{\underbrace{s_1,\ \ldots,\ s_t}_{\tilde{s}},\ s_{t+1}}^{s^L},\ \underbrace{\phantom{s_{t+1},}\ s_{t+2},\ \ldots,\ s_n}_{s^R}\right) \neq s^R$$

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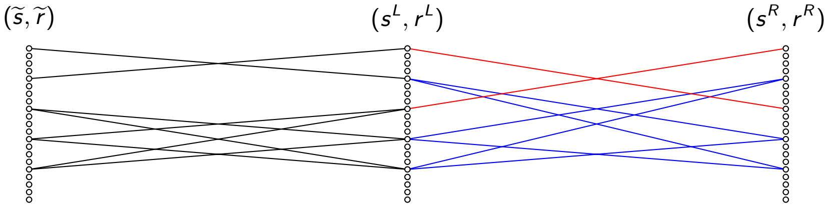

$\mathrm{Share}(\widetilde{s}, \widetilde{r})_{\{1,\ldots,t\}} = \mathrm{Share}(s^L, r^L)_{\{1,\ldots,t\}}$ $\quad$ $\mathrm{Share}(s^L, r^L)_{t+1} = \mathrm{Share}(s^R, r^R)_{t+1}$

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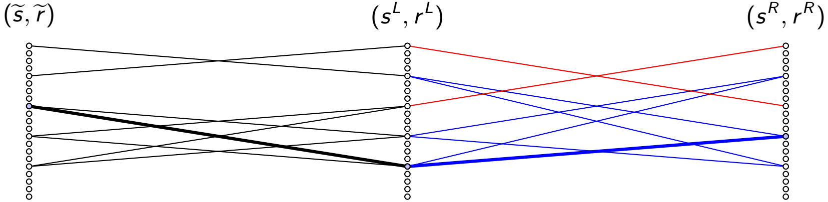

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$$\delta = 2^{-k} \geq \Pr_{(\widetilde{s},\widetilde{r},s^R,r^R)}[\exists (s^L, r^L) \mid (\widetilde{s}, \widetilde{r}) — (s^L, r^L) \overset{\text{L}}{—} (s^R, r^R)]$$

Mass Facts

For $a_1, \ldots, a_{t+1}$,
let $B_{a_1,\ldots,a_{t+1}} = \{(s^L, r^L) \mid \text{Share}(s^L, r^L)_{\{1,\ldots,t+1\}} = a_1, \ldots, a_{t+1}\}$,
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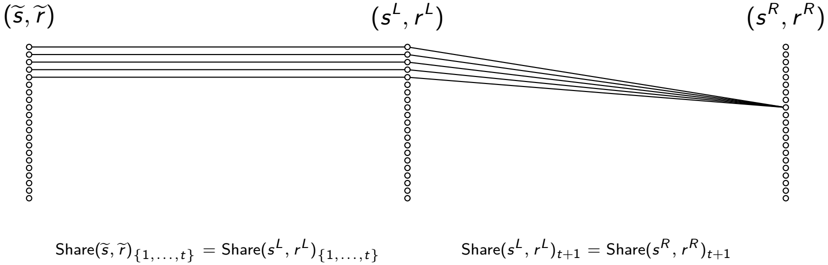
Fact 1*: by reconstructability, $(s', r'), (s'', r'') \in B_{a_1,\ldots,a_{t+1}} \Rightarrow s' = s''$.



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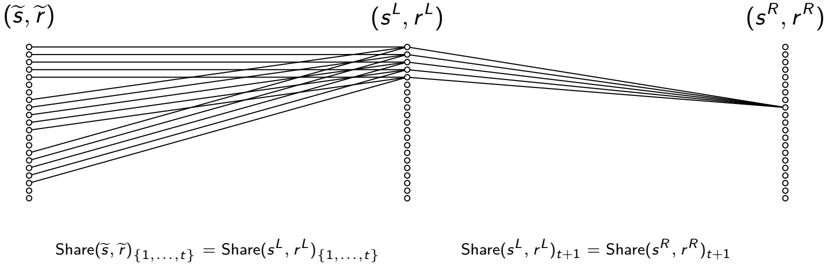
Fact 1*: by reconstructability, $(s', r'), (s'', r'') \in B_{a_1,\ldots,a_{t+1}} \Rightarrow s' = s''$.
Fact 2: by privacy, $|A_{a_1,\ldots,a_{t+1}}| \geq 2^m \cdot |B_{a_1,\ldots,a_{t+1}}|$.



Putting Things Together – Intuition

Actual analysis needs more correcting factors (loss of ~ 4 bits).

$$2^{-k} \geq \Pr_{(\widetilde{s},\widetilde{r},s^R,r^R)}[\exists(s^L, r^L) \mid (\widetilde{s},\widetilde{r}) \text{—} (s^L, r^L) \overset{\text{L}}{\text{—}} (s^R, r^R)]$$

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$$\geq 2^m \cdot \Pr_{(s^L,r^L,s^R,r^R)}[(s^L, r^L) \overset{\mathsf{L}}{\text{—}} (s^R, r^R)]$$

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$$
\begin{aligned}
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&\geq 2^m \cdot \Pr_{(s^L,r^L,s^R,r^R)}[(s^L, r^L) \overset{\mathsf{L}}{\text{—}} (s^R, r^R)] && (\#\mathsf{L} \geq \#\mathsf{R}) \\
&\geq 2^{m-1} \cdot \Pr_{(s^L,r^L,s^R,r^R)}[(s^L, r^L) \text{—} (s^R, r^R)] \\
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&\geq 2^{m-1} \cdot \sum_{a_{t+1}} \Pr_{(s,r)}[\mathsf{Share}(s, r) = a_{t+1}]^2
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Putting Things Together – Intuition

Actual analysis needs more correcting factors (loss of ~ 4 bits).

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2^{-k} &\geq \Pr_{(\widetilde{s},\widetilde{r},s^R,r^R)}[\exists (s^L,r^L) \mid (\widetilde{s},\widetilde{r}) — (s^L,r^L) \overset{\text{L}}{—} (s^R,r^R)] && \text{(Fact 1\&2)}\\
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Conclusion

Robust SS with $n = 2 \cdot t + 1$ players, eff. reconstruction. Share size:

model	construction	lower bound
standard	$m + \widetilde{O}(n + k)$	$m + k$
NEW: local adv.	$m + \widetilde{O}(k)$	$m + k - 4$

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THANKS!

`https://eprint.iacr.org/2014/909`

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